Abstract

Binary trees come in two varieties: plane trees, often simply called binary trees, and non-plane trees, in which the order of subtrees does not matter. Non-plane trees find many applications; for example in modeling epidemics, in studying phylogenetic trees, and as models in data compression. While binary trees have been studied very extensively, non-plane trees still pose some challenges. Moreover, in most analyses a uniform probabilistic model is assumed; that is, a tree is selected uniformly from among all trees. Such a model limits significantly applications of the analysis. In this paper we study by analytic techniques non-plane trees in a non-uniform model. In our model, we grow the tree on n leaves by selecting randomly a leaf and appending two children to it. We can show that this is equivalent to an alternative model, also used to analyze the average-case performance of binary search trees, that is more easily amenable to study by recurrences and generating functions. Here, one of the most important questions is the number of symmetries in such trees (i.e., the number of internal nodes with two isomorphic subtrees), or the sizes of such symmetric subtrees. We first present a functional-differential equation characterizing tree symmetries, and then analyze it. In this conference paper we focus on the expected number of symmetries, the size of symmetric subtrees, and the tree entropy.