Abstract

Panagiotou and Stufler (arXiv:1502.07180v2) recently proved one important fact on their way to establish the scaling limits of random Pólya trees: a uniform random Pólya tree of size n consists of a conditioned critical Galton-Watson tree C_n and many small forests, where with probability tending to one as n tends to infinity, any forest $F_n(v)$, that is attached to a node v in C_n, is maximally of size $|F_n(v)| = O(\log n)$. Their proof used the framework of a Boltzmann sampler and deviation inequalities. In this paper, first, we employ a unified framework in analytic combinatorics to prove this fact with additional improvements on the bound of $|F_n(v)|$, namely $|F_n(v)| = \Theta(\log n)$. Second, we give a combinatorial interpretation of the rational weights of these forests and the defining substitution process in terms of automorphisms associated to a given Pólya tree. Finally, we derive the limit probability that for a random node v the attached forest $F_n(v)$ is of a given size.