A System of Equations

Problem 07-006, *by* APOLONIUSZ TYSZKA[1] (Technical Faculty, Hugo Kołłątaj University, Kraków, Poland).

Prove or disprove the following conjecture: given $n \geq 1$ and $x_1, x_2, \ldots, x_n \in \mathbb{R}$ ($\mathbb{C}$) there exists $y_1, y_2, \ldots, y_n \in \mathbb{R}$ ($\mathbb{C}$) for all $i, j, k \in \{1, 2, \ldots, n\}$,

$$x_i = 1 \Rightarrow y_i = 1,$$

$$x_i + x_j = x_k \Rightarrow y_i + y_j = y_k,$$

$$x_i \cdot x_j = x_k \Rightarrow y_i \cdot y_j = y_k, \text{ and}$$

$$|y_i| \leq 2^{2^{n-2}}.$$

Comments. Alternatively, given a system of equations in real or complex numbers $x_1, x_2, \ldots, x_n$, where each equation in the system is one of the three forms (a) $x_i = 1$, (b) $x_i + x_j = x_k$, or (c) $x_i \cdot x_j = x_k$, if the system is consistent, then it has a solution in which $|x_i| \leq 2^{2^{n-2}}$ for each i. In [1] the proposer gives a proof of the conjecture for $n \leq 3$.

Status. This problem is open.

REFERENCE

[1] A. TYSZKA, *Bounds of some real (complex) solution of a finite system of polynomial equations with rational coefficients*, submitted, `http://arxiv.org/abs/math/0702558`.

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