

How Smooth Should f Be So That $1/f$ Is Convex?

Problem 07-004, by SLAVKO SIMIC¹ (Mathematical Institute SANU, Belgrade, Serbia).

It is said that $f \in C_I^\infty$ has *smoothness* of order n if the first n derivatives of f are strictly positive on $I := [0, \infty)$. Is there a finite order of smoothness of f that implies that $1/f(x)$ is convex for all sufficiently large x ? If so, find the smallest such n .

Editorial Note. Partial solutions are welcome. For example, the proposer has an example where f is positive, increasing and convex, but $1/f$ is not convex. Additional counterexamples for specific orders of smoothness would be appreciated.

Status. This problem is open.

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