A Gaussian Integer Zeta Sum

Problem 05-002, *by* Jonathan Borwein (Dalhousie University, Halifax, NS, Canada).

Evaluate in closed form

$$\zeta_G(N) = \sum_{z\in\mathbb{Z}[i]}{}' \frac{1}{z^N} = \sum_{m,n}{}' \frac{1}{(m+in)^N}$$

for positive integer $N > 1$. Here, as always, the primed summation signifies that $(m, n) = (0, 0)$ is excluded, so we sum over all nonzero Gaussian integers. Here "closed form" means the product of a simply determined rational number and a power of a special function value.

Status. The proposer has a solution. Additional solutions are welcome.