Solution of Problem 02-002 *by the proposer.* Let R_k denote the driving-point resistance between two vertices a distance k apart in Q_k. Suppose a unit current flows into the network Q_{n+1} at vertex $A = (1, 1, \ldots, 1)$ and flows out from $B = (0, 0, \ldots, 0)$. Identify the cube with the Hasse diagram of subsets of $\{1, 2, \ldots, n+1\}$ ordered by inclusion. Say that vertices that correspond to k-element subsets are at *level* k in the network. By symmetry, all $\binom{n+1}{k}$ vertices at level k are at the same potential, so they may be coalesced into one vertex without changing externally observed electrical properties of the cube. Thus Q_{n+1} may be replaced by a series-parallel network with $n+2$ vertices $0, 1, \ldots, n+1$. Vertex k of the series-parallel network is obtained by coalescing all cube vertices at level k. The number of parallel edges joining vertex k to vertex $k+1$ is

$$\binom{n+1}{k}(n+1-k) = \binom{n+1}{k+1}(k+1) = (n+1)\binom{n}{k}, \qquad 0 \le k \le n.$$

Hence the resistor joining vertices k and $k+1$ in the equivalent network has resistance $\frac{1}{n+1}\binom{n}{k}^{-1}$, and therefore

$$R_{n+1} = \frac{1}{n+1}\sum_{k=0}^{n}\binom{n}{k}^{-1}.$$

The basic idea of this part of the argument is illustrated (for $n = 2$) by the following figure.

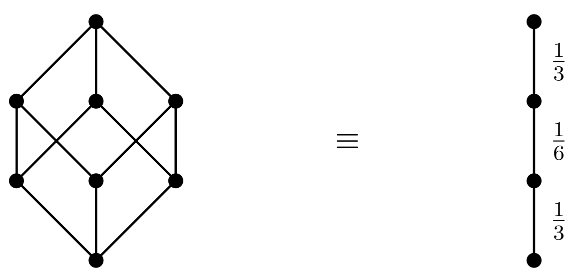

FIG. 1 - *Method* 1 *for Computing the Driving-Point Resistance.*

To compute R_{n+1} a second way, we follow Rennie's approach in [4]. This makes use of symmetry properties of the cube and the principle of superposition. Let $C = (1, 1, \ldots, 1, 0)$ and $D = (0, 0, \ldots, 0, 1)$. If a unit current enters at A and exits at B (ground), then by symmetry, the potentials at A, B, C, D are

$$v(A) = R_{n+1}, \quad v(B) = 0, \quad v(C) = R_{n+1} - \frac{1}{n+1}, \quad v(D) = \frac{1}{n+1}.$$

In the same way, if a unit current enters at C and exits at D (ground), the potentials are

$$v(A) = R_{n+1} - \frac{1}{n+1}, \quad v(B) = \frac{1}{n+1}, \quad v(C) = R_{n+1}, \quad v(D) = 0.$$

By the principle of superposition, if unit currents enter at A and C and exit at B and D, then

$$v(A) - v(B) = v(C) - v(D) = 2R_{n+1} - \frac{2}{n+1}.$$

Now we use the fact that Q_{n+1} consists of two copies of Q_n together with edges joining corresponding vertices. In particular Q_{n+1} is so formed from the two n-dimensional cubes induced by $\{(x_1, x_2, \dots, x_n, 0)|\ x_i \in \{0,1\}\}$ and $\{y_1, y_2, \dots, y_n, 1)|\ y_i \in \{0,1\}\}$. It is easy to see by symmetry that if unit currents enter at A and C and exit at B and D, there is no current flow along the edges joining corresponding vertices of the aforementioned copies of Q_n. In particular $v(A) = v(C)$ and $v(B) = v(D)$. For the same reason, it is just as if corresponding vertices in the two copies were not joined at all. Hence $v(A) - v(B) = v(A) - v(C) = R_n$. Hence $R_n = v(A) - v(B) = 2R_{n+1} - 2/(n+1)$. Thus we have the recurrence

$$2^{k+1}R_{k+1} - 2^k R_k = \frac{2^{k+1}}{k+1}, \quad k \geq 0 \qquad (R_0 = 0),$$

which gives

$$2^{n+1}R_{n+1} = \sum_{k=0}^{n}(2^{k+1}R_{k+1} - 2^k R_k) = \sum_{k=0}^{n} \frac{2^{k+1}}{k+1}.$$

The basic idea of this part of the argument is illustrated (for $n = 2$) by the following figure.

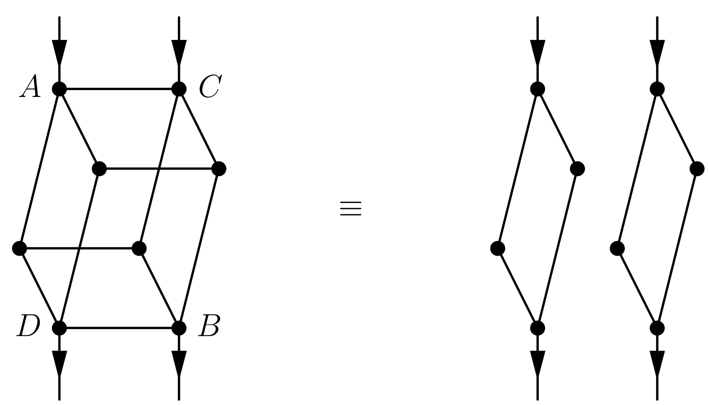

FIG. 2 - *Method 2 for Computing the Driving-Point Resistance.*

Comparing the two results for R_{n+1}, we find

$$\frac{1}{n+1}\sum_{k=0}^{n}\binom{n}{k}^{-1} = \sum_{k=0}^{n}\frac{2^{k-n}}{k+1} = \sum_{k=0}^{n}\frac{1}{2^k(n+1-k)},$$

so

$$\sum_{k=0}^{n}\binom{n}{k}^{-1} = (n+1)\sum_{k=0}^{n}\frac{1}{2^k(n+1-k)}.$$