A Simple Solution of a Problem of Korman

Solution of Problem 00-001 *by* Michael Renardy (Virginia Polytechnic Institute and State University).

The necessity of the condition follows by integrating the differential equation over the interval [0,1]: If there is a solution u, then

$$f(\infty) < \int_0^1 f(u(x))\,dx = \int_0^1 g(x)\,dx < f(-\infty).$$

To see the sufficiency, consider the initial value problem

$$u'' + f(u) - g(x) = 0, \qquad u(0) = a,\ u'(0) = 0.$$

It is easy to see that, as $a \to \infty$,

$$u'(1) = \int_0^1 u''(x)\,dx = \int_0^1 \{g(x) - f(u(x))\}\,dx \to \int_0^1 g(x)\,dx - f(\infty).$$

Similarly,

$$u'(1) \to \int_0^1 g(x)\,dx - f(-\infty)$$

as $a \to -\infty$. If (3) holds, then the intermediate value theorem implies the existence of a value a for which $u'(1) = 0$.